5.3 Exponential Models

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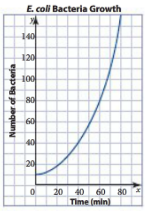
Example 1 p.296
Reading Values of a Graph

Growth of Bacteria

When some foods, such as fresh meat, are left at room temperature, any bacteria in the food will begin to multiply. If enough bacteria are present, you can become ill from eating the food. This is known as food poisoning. The most common bacteria found in food are salmonella and *E. coli*. The graph shows the growth of an *E. coli* bacteria population over time.



a) Describe the relationship between number of bacteria and time.

b) Use the graph to estimate the number of bacteria after each time period.
i) 20 min ii) 40 min iii) 60 min

c) Calculate the ratios. Divide the number of bacteria after 40 min by the number after 20 min. Divide the number of bacteria after 60 min by the number after 40 min.

d) What happens to the number of bacteria every 20 min?

e) Consider the rate of change of number of bacteria with respect to time. What are suitable units for this rate of change?

f) Is the rate of change of number of bacteria with respect to time increasing, constant, or decreasing? Justify your answer.

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5.3 Exponential Models

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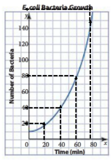
Example 1 p.296
Reading Values of a Graph

Growth of Bacteria

When some foods, such as fresh meat, are left at room temperature, any bacteria in the food will begin to multiply. If enough bacteria are present, you can become ill from eating the food. This is known as food poisoning. The most common bacteria found in food are salmonella and *E. coli*. The graph shows the growth of an *E. coli* bacteria population over time.



a) Describe the relationship between number of bacteria and time.
→ multiply

b) Use the graph to estimate the number of bacteria after each time period.
i) 20 min 20 ii) 40 min 40 iii) 60 min 80

c) Calculate the ratios. Divide the number of bacteria after 40 min by the number after 20 min. Divide the number of bacteria after 60 min by the number after 40 min.

d) What happens to the number of bacteria every 20 min? doubles

e) Consider the rate of change of number of bacteria with respect to time. What are suitable units for this rate of change? # of bacteria/min

f) Is the rate of change of number of bacteria with respect to time increasing, constant, or decreasing? Justify your answer.

Δy Ratio Δy'

		Δy	Ratio
20	20	20	40/20 → 2
40	40	40	80/40 → 2
60	80	80	160/80 → 2
80	160		

constant ratio

Increasing / Doubling

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ii) Graphing Exponential Relationships

Example 2 using TI83 p. 297

Smoke Detectors

Americium-241 (Am-241) is a manufactured element. It is a silvery radioactive metal, which is used in smoke detectors. Household smoke detectors contain about 200 micrograms (μg) of Am-241. The amount of Am-241 present in the detector decreases or decays over time. The table shows the mass of Am-241 remaining, in micrograms, over 1000 years.

Years	Mass Remaining (μg)
0	200
100	170
200	145
300	124
400	105
500	90
600	76
700	65
800	55
900	47
1000	40

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a) Does the Years column show equal time intervals?

b) Calculate the first and second differences. Is the relationship between mass remaining and time linear? quadratic? Explain how you know.

c) Calculate the ratios. Divide the second mass by the first, the third by the second, and so on. Is the relation exponential? Explain how you know.

d) Draw a graph with years on the horizontal axis and mass remaining on the vertical axis. Can you use the graph to rule out a linear relation? a quadratic relation? Justify your answer.

e) Compare the table and the graph. Does the rate of change of mass remaining with respect to years appear to be constant, increasing, or decreasing? Justify your answer.

f) What are suitable units for the rate of change of mass remaining with respect to years?

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a) Does the Years column show equal time intervals? yes

b) Calculate the first and second differences. Is the relationship between mass remaining and time linear? quadratic? Explain how you know.

c) Calculate the ratios. Divide the second mass by the first, the third by the second, and so on. Is the relation exponential? Explain how you know.

d) Draw a graph with years on the horizontal axis and mass remaining on the vertical axis. Can you use the graph to rule out a linear relation? a quadratic relation? Justify your answer.

e) Compare the table and the graph. Does the rate of change of mass remaining with respect to years appear to be constant, increasing, or decreasing? Justify your answer.

f) What are suitable units for the rate of change of mass remaining with respect to years?

Years	Mass	First diff.	Second diff.
0	200	-30	+5
100	170	-25	+4
200	145	-21	+2
300	124	-19	+4
400	105	-15	
500	90		

c) $\frac{170}{200} = 0.85$ ∴ Exponential

$\frac{145}{170} = 0.85$

$\frac{124}{145} = 0.86$

Decay b/c 1 > x

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iii) Exponential Regression

Example 3 p. 299

Draw Cards

Sandor drew cards from a standard deck of 52 playing cards until he drew a heart. He repeated this experiment many times. Each time, Sandor recorded the number of cards he drew before drawing a heart. For example, if he drew a heart on the first draw, he drew zero cards before he drew a heart; this happened 50 times.

Number of Cards	Frequency
0	50
1	38
2	28
3	21
4	16
5	12
6	9

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a) Calculate the first and second differences and the ratios between consecutive pairs of frequencies. Does the relationship between frequency and number of cards appear to be linear? quadratic? exponential? Explain how you know.

b) Use technology to fit an exponential relation to the data. Write an equation for the relation.

c) Use the equation from part b) to predict the frequency of drawing eight cards before drawing a heart.

d) Draw a graph with the number of cards on the horizontal axis and the frequency on the vertical axis.

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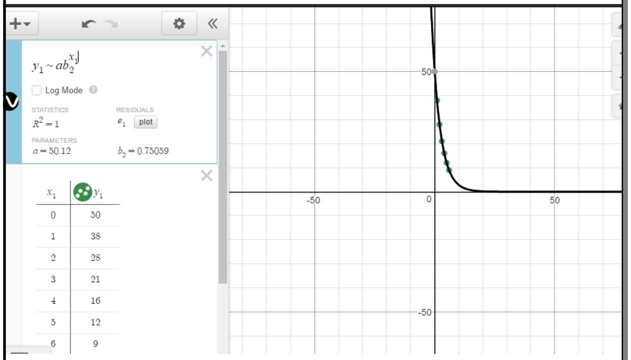

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Solution

a) The first differences are not constant, so the relationship between frequency and number of cards is non-linear. The second differences are not constant, so the relation is not quadratic. The ratios are approximately constant. The relationship between number of cards and frequency could be modelled using an exponential relation.

Number of Cards	Frequency	First Differences	Second Differences	Ratios
0	50			
1	38	−12		0.76
2	28	−10	2	0.74
3	21	−7	3	0.75
4	16	−5	2	0.76
5	12	−4	1	0.75
6	9	−3	1	0.75

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Key Concepts

- *If ratios are constant , the relation is exponential*
- *In an exponential relation, there is a constant percent increase over equal intervals*

$y_1 \sim ab^{x_1}$ $\boxed{a^b}$

Regression Equation

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q. 1-4, 6-8 9*

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